CHESAPEAKE BAY INSTITUTE
The Johns Hopkins University

A NEW SET OF OCEANIC DIFFUSION DIAGRAMS

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Akira Okubo

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Chesapeake Bay Institute
The Johns Hopkins University

A NEW SET OF
OCEANIC DIFFUSION DIAGRAMS

Akira Okubo

TECHNICAL REPORT 38

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D. W. Pritchard
Director

ABSTRACT

Some empirical relations between diffusion characteristics are investigated by the use of carefully examined data from dye release experiments in the surface layer of the sea. These data cover a time scale of diffusion ranging from 1 hour to 1 month and a length scale from 100 m to 100 km. Two "oceanic diffusion diagrams" are prepared; one showing horizontal variance versus diffusion time and the other showing apparent diffusivity versus the scale of diffusion. The overall behaviors of the horizontal variance and of the apparent diffusivity are evidently different from those which the similarity theory of turbulence deduces. However, there still remains a possibility that the similarity theory may be valid locally in time- or length-scale with a variable parameter, i.e., the rate of turbulent energy transfer. The diagrams provide a practical means to predict the rate of horizontal spread of substance introduced from an instantaneous point-source as well as the apparent diffusivity as a function of the size of diffusing patch.

ACKNOWLEDGMENTS

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Some of the unpublished data of the dye experiments used in this report were supplied to the author through the courtesy of other investigators; the shipboard data of the operation "RHENO" and 1964-release in the North Sea were provided by Drs. H. Weidemann and H. Sendner of Deutsches Hydrographisches Institut, Hamburg, and the detailed information on the dye experiments off Southern California by Dr. J. E. Foxworthy of Allan Hancock Foundation, University of Southern California. The author wishes to express special appreciation to them.

TABLE OF CONTENTS

ILLUSTRATIONS

ILLUSTRATIONS (Cont'd)

TABLES

1. Introduction. Turbulent diffusion of contaminants is not only a matter of theoretical interest but is also a problem of practical importance in connection with the discharge of industrial wastes including radioactive materials into the oceans. The process of oceanic diffusion is quite complex, and hardly any single theory can explain or interpret the entire pattern of the diffusion. In addition to theoretical studies, therefore, progress in oceanic diffusion is still very much dependent on an empirical approach by means of diffusion experiments.

Among various types of tracer used in experimental studies of oceanic diffusion, fluorescent dyes have proved the most promising, especially Rhodamine B. It has been regarded as one of the most accessible, stable, harmless, and convenient fluorescent tracers for use in sea experiments (Carpenter, 1960, Okubo et al 1957).

Since Pritchard and Carpenter (1960) developed a new field technique for the direct continuous observation of the fluorescent concentration with the aid of a fluorometer, experimental data of oceanic diffusion have been accumulated. In particular, an internationally cooperative experiment, operation "RHENO", in the North Sea made it possible to cover a scale of diffusion ranging up to 100 km and a time scale up to one month. Toward the other end of the scale, a set of experiments off the coast of Southern California provided the data of small scale diffusion of 30 m or so. A fairly large amount of data have been obtained for intermediate scales of diffusion ranging, say from 1 km to 10 km. So far no appropriate data are available from dye experiments in the deep layer. Hence we are restricted to data obtained in the surface layer.

Using the available data of horizontal diffusion until 1962, Okubo (1962) presented a set of diagrams showing the empirical relations between the variance of the concentration distribution and diffusion time and between the coefficient of eddy diffusion and the scale of diffusion.

In this report, a new set of diffusion diagrams are constructed on the basis of new data accumulated since 1961. None of the data chosen here was used in the diffusion diagrams constructed in 1962 in order to obtain completely new diagrams. Carefully examined data were analysed by a consistent method in order to compute the variance of the horizontal distribution of concentration. One of the purposes of the present study is to compare the findings in the old diffusion diagrams with the new set of diagrams.

2. Data used in the present study. After exhaustive search the author has found 20 sets of diffusion experiments, which provide data suitable for the present purpose. The criteria for defining suitable data are as follows:

1) The release of dye should be as close to a type of instantaneous, point source as possible; the duration of release and the initial (horizontal) size of dye patch must be reported or somehow estimated at least to an order of magnitude. For a known initial size or more precisely a known initial variance, a critical time after which the diffusion is regarded practically as from a point source can be estimated from a theoretical relationship for the rate of change of variance, e.g., the theory of Joseph and Sendner(1958) gives

$$\sigma_o^2 = 6\, P^2 t_o^2 \quad , \tag{1}$$

where σ_o^2 : the variance associated with the initial distribution of patch, P: a diffusion velocity after Joseph and Sendner, and t_o: a characteristic time of diffusion of patch associated with the initial size, σ_o . Obviously the critical time must be much larger than t_o. An arbitrary choice of the critical time herein made is 10 times of t_o. Diffusion data prior to the critical time are either disregarded or corrected to an appropriate diffusion time by adding the critical time to the actual diffusion time.

2) The dye patch should stay a sufficient distance from vertical boundaries so that the field of diffusion may be regarded as extending to infinity in the horizontal direction. In most cases whether a dye patch has felt the boundaries or not can be determined intuitively from the observed pattern of concentration distribution. One should also be concerned with the effect of the bottom boundary so that the diffusion in question may be classified as two-dimensional or three-dimensional. For this purpose, a sharp thermocline is regarded as a boundary. The presence or absence of constraints in the vertical direction does not create a drastic change in horizontal diffusion but have a significant influence on the time behavior of the maximum concentration of patch.

3) The horizontal distribution of dye concentration should be observed in such a manner so that the horizontal variance associated with the dye distribution can be computed directly. Thus aerial photographs of the dye patch are not considered useful for this purpose, although some progress has been made in determining the concentration distribution from dye photos (Ichiye and Plutchak, 1966). In observing the time change of patch size in photographs, Gifford (1957) proposed an indirect method to estimate the variance in the case of atmospheric diffusion. Ichiye (1959) and Okubo (1962) applied this technique, seemingly with some success, to horizontal diffusion of dye in the sea. However, Gifford's technique is not employed in this report, not only because the direct computation of the variance is more advantageous but also because the variance estimated by means of the Gifford technique depends, to some extent, on the choice of the assumed distribution for dye concentration.

4) The concentration distribution should agree with a "mass or material balance", by which is meant that the amount of dye remaining within a patch must be comparable to the total amount of dye released, when the loss of dye due to photochemical decay, physical adsorption, etc., if appreciable, is corrected. Pritchard

and Carpenter (1960) in their study of diffusion in Chesapeake Bay, reported that mass balances of 80 to 90% were realized using Rhodamine B as a tracer. This case is, however, a little ideal. Usually mass balances as low as 50% or as high as 150% may be taken as permissible especially in open ocean experiments and in the later stage of diffusion characterized by very low concentrations of dye within a patch. Quite often the mass balance is neither reported nor computable simply because of lack of information on the vertical distribution of dye. In this case an appropriate value of the mixed layer within which dye is assumed to be distributed uniformly is taken on the basis of some indirect information, say the stratification of the water column. The amount of dye in a patch is then computed from the horizontal distribution of dye multiplied by the depth of the mixed layer. Another way of examining the mass balance represents an indirect approach: take the observed horizontal distribution of dye and compute the depth of mixed layer within which 100% of material is supposed to remain. If this fictitious depth turns out to be reasonable from the standpoint of the knowledge of vertical diffusion in the sea, the stratification of the water column, the total depth of water, etc., we may regard the experiment as "good" with respect to a mass balance.

3. Computation of diffusion characteristics. A diffusion diagram consists of a plot of a characteristic property of diffusion against an another such property. A set of the most natural choices for characteristic properties is the variance of the horizontal distribution* of material versus the diffusion time, i.e., the time elapsed since the

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* The maximum concentration, for example, may not be a good measure of diffusion, not only because the observation of the maximum concentration involves a great deal of uncertainty but also because the peak concentration is very sensitive to the presence or absence of a horizontal boundary.

introduction of material. Usually the origin of the time (the "release time") is defined as the midpoint of the time of release, and the diffusion time is the difference between the midpoint of the time spent in observing the concentration distribution and the release time. The part of the concentration distribution observed prior to a diffusion time would be diffused, to some extent, if one observed it instantaneously at the diffusion time. On the contrary, the other part of the distribution observed after the diffusion time would be less diffused if observed instantaneously at the diffusion time. Thus the net effect is compensatory, so that a more or less instantaneous value of the variance may be obtained with respect to the diffusion time.

The variance of the horizontal distribution is certainly a suitable measure of the spread of the contaminant. Horizontal diffusion in the sea can not be described adequately by a Fickian-type diffusion with a constant coefficient of diffusion. Thus the theory of oceanic diffusion has been directed toward non-Fickian diffusion; as a matter of fact, the apparent diffusivity increases with the diffusion time or with the size of the diffusive substance (Okubo, 1962).

In order to estimate the variance directly, one must obtain a fairly good pattern of the horizontal concentration distribution at a particular depth. An example of such a concentration distribution is shown in Fig. 1, where we also indicate the ship tracks along which continuous recordings of concentration are made. Since the horizontal distribution of substance is usually asymmetric in the sense that a characteristic length is larger in one direction or another, two variances, along the long and short axes, say, are necessary to properly describe the rate of dispersion. However, conversion to a radially symmetric equivalent distribution has been widely used in reporting the results of dye experiments.

The concept involved in a radially symmetric patch is as follows. At a diffusion time t, the shape of the isolines of any concentration will be very irregular in any individual case; the patch of

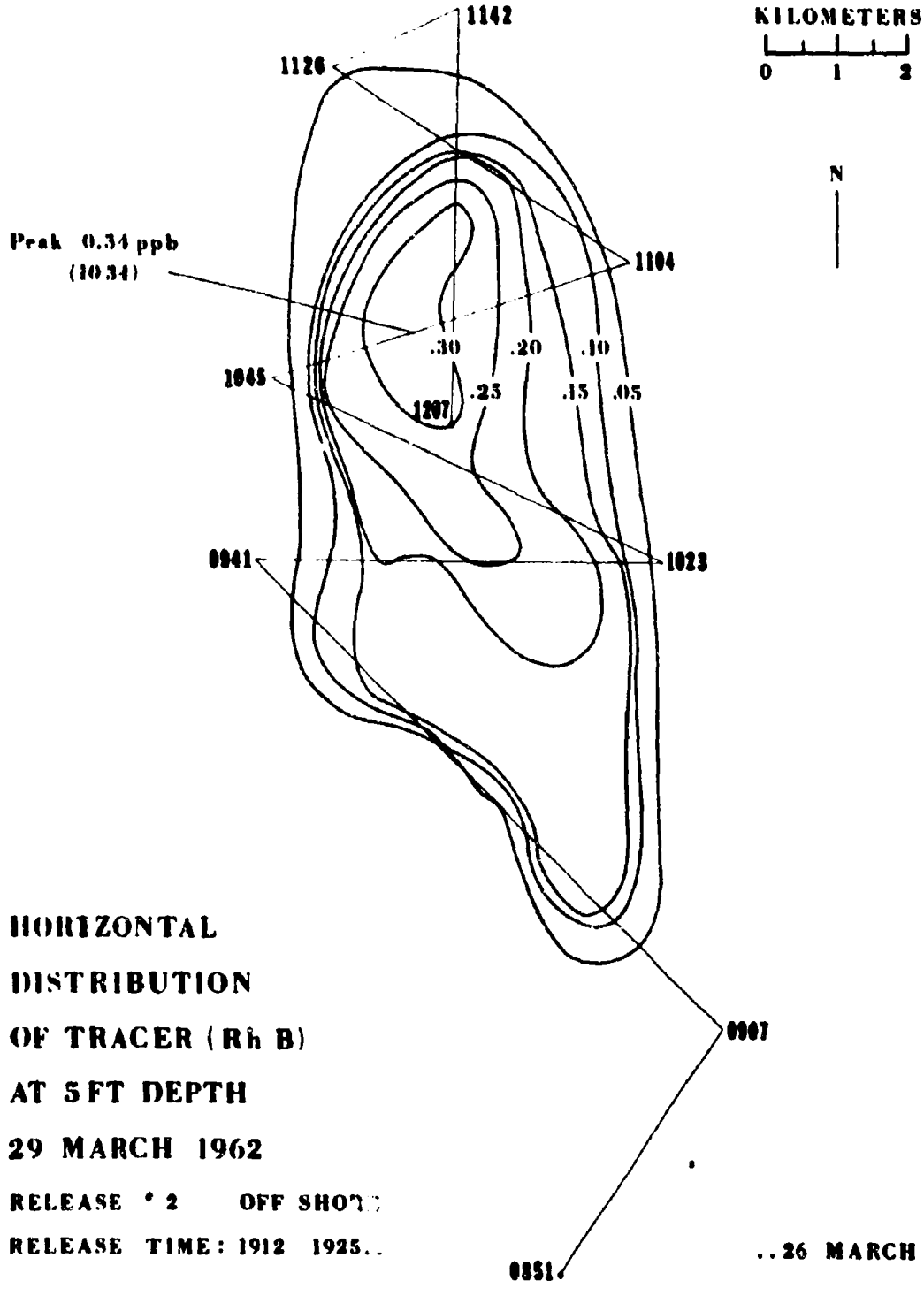


Fig. 1. Horizontal distribution of rhodamine B at 5 ft depth for release #2 (off Cape Kennedy).

substance is often elongated. We shall now carry out (conceptually!) an infinite number of releases under identical oceanographic conditions, using in each case the same amount of substance released. Thus, if we take an infinite number of distributions each observed at the same diffusion time, if we superpose them in such a way that the center of mass coincides with each other, and if we average over all the superposed distributions, we would then expect a radially symmetric distribution of substance about the center of mass, provided that the oceanic conditions, e.g., the shears in the mean current, are isotropic.

Normally identical oceanic conditions do not necessarily imply rotational symmetry in the shears. A case in which the shears in the mean flow are likely to occur predominantly in one direction is seen in the nearshore region. There almost always a patch of dye tends to elongate in the direction parallel to the coast. Thus the ensemble average of distribution would be more or less elongated rather than radially symmetrical. A radially symmetrical distribution is, however, the simplest mathematical model to deal with. Thus the variance associated with a radially symmetric distribution can be related through a measure of the eccentricity to the variances for a two-dimensional elliptical distribution which itself is a proper mathematical model for an elongated patch of dye (Okubo, 1965). The Appendix summarizes the relationships between various variances associated with particular mathematical models for the two-dimensional distribution of substance.

Intuition tells us that the area enclosed by a line of constant concentration would be nearly the same for all identical releases in spite of the very irregular distribution of concentration in individual releases. As a measure of the area enclosed by a particular concentration, we define the radius r_e of a circle of equal area. A radially symmetrical distribution, $S(t, r_e)$, thus characterized by the equivalent radius and the diffusion time would represent

approximately the radially symmetric distribution in the ensemble average mentioned previously.

Having obtained a sufficient number of data on r_e vs. S for a patch of dye, one may compute the variance σ_{rc}^2 by the definition.

$$\sigma_{rc}^2(t) \equiv \int_0^\infty r_e^2 \, S(t,r_e) \, 2\pi r_e \, dr_e \Big/ \int_0^\infty S \, 2\pi r_e \, dr_e \quad . \tag{2}$$

Frequently the spatial distribution of S is well described by a two-dimensional Gaussian distribution, i.e.,

$$S(t,r_e) = \frac{M/D}{\pi\sigma_{rc}^2(t)} \, e^{-\frac{r_e^2}{\sigma_{rc}^2(t)}} \quad , \tag{3}$$

where M/D denotes the total mass of dye per unit depth of mixed layer. In this case σ_{rc}^2 can be estimated from a plot of log S vs. r_e^2. Fig. 2 shows such an example.

In another case S(t, r_e) can be described by a generalized exponential distribution:

$$S(t,r_e) = S(t,o) \, e^{-\left(\frac{r_e}{\sigma_{rc}(t)}\right)^m} \quad , \tag{4}$$

where S(t, o) is the peak concentration and m is a positive number.

Thus a plot of $\log \frac{S(t,o)}{S(t,r_e)}$ vs. r_e will determine the value of σ_{rc}^2 as well as m. An example is seen in Fig. 3. In any case, we can calculate σ_{rc}^2 directly from definition (2).

The accuracy of the estimate of σ_{rc}^2 depends primarily on uncertainties in the equivalent radii. "Good" data by which is meant that a fairly precise distribution of concentration is observed, should provide an estimate of σ_{rc}^2 within ± 5% of accuracy. Completely

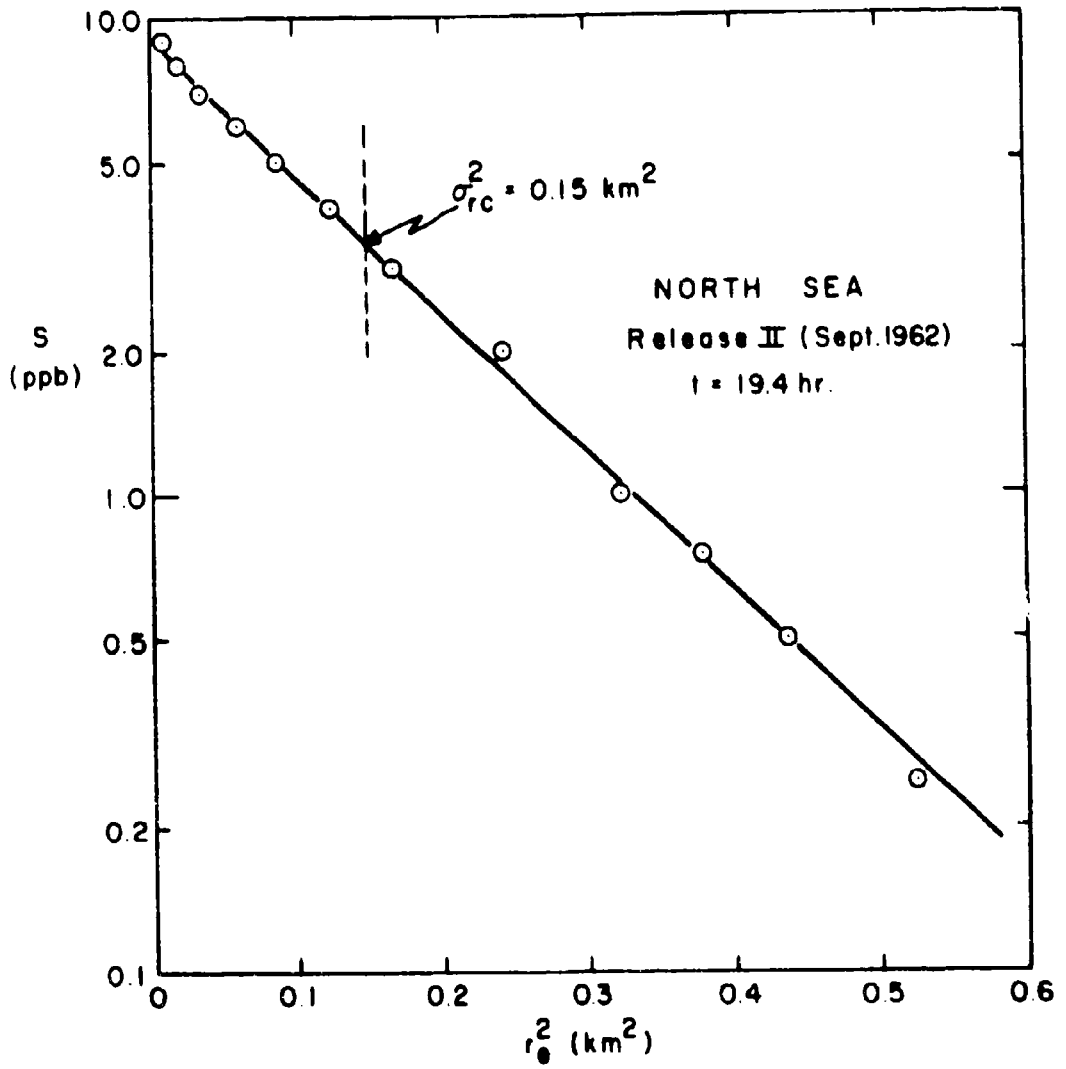


Fig. 2. Fit of diffusion data to a Gaussian distribution.

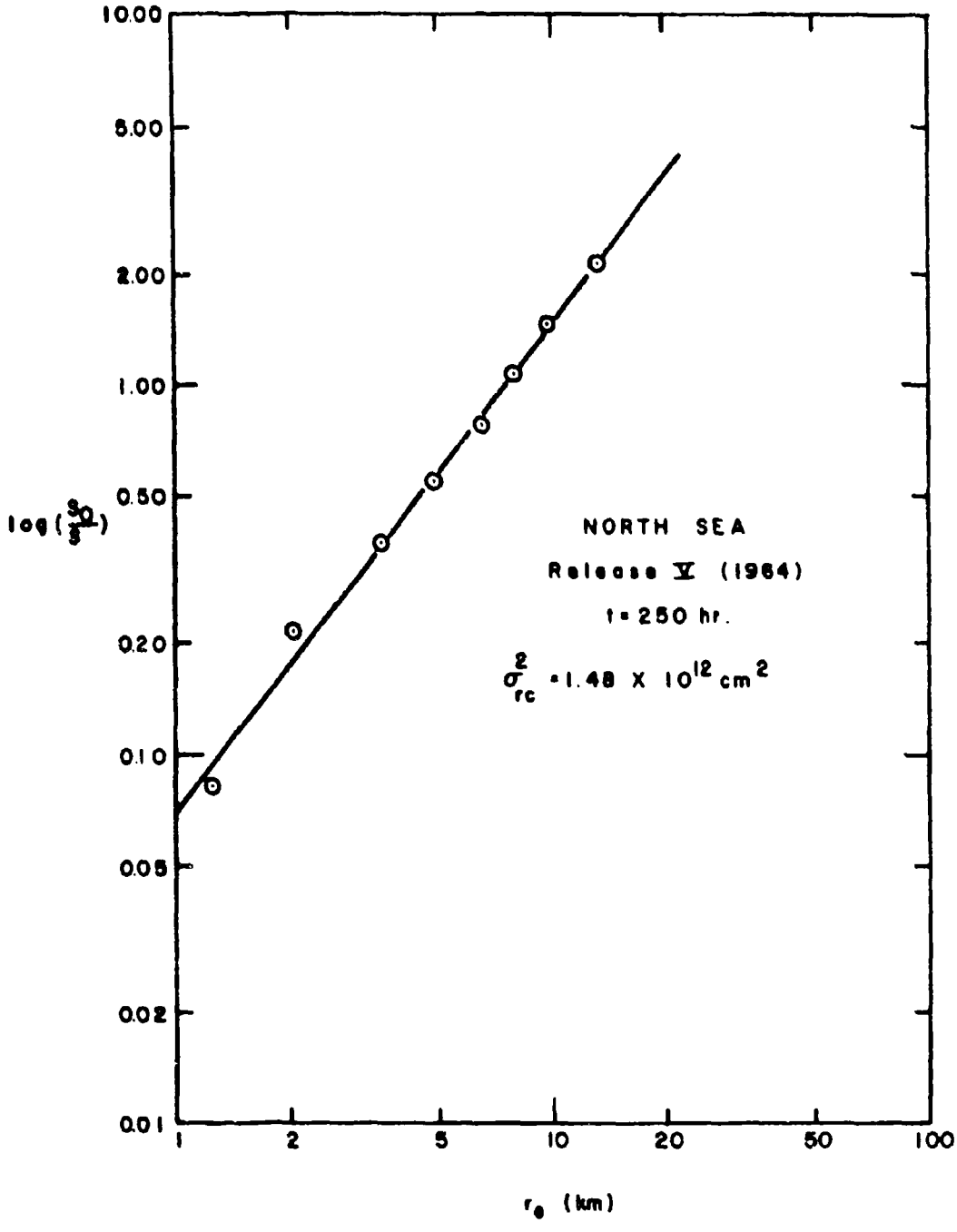
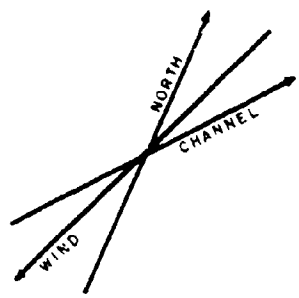


Fig. 3. Fit of diffusion data to a generalized exponential distribution.

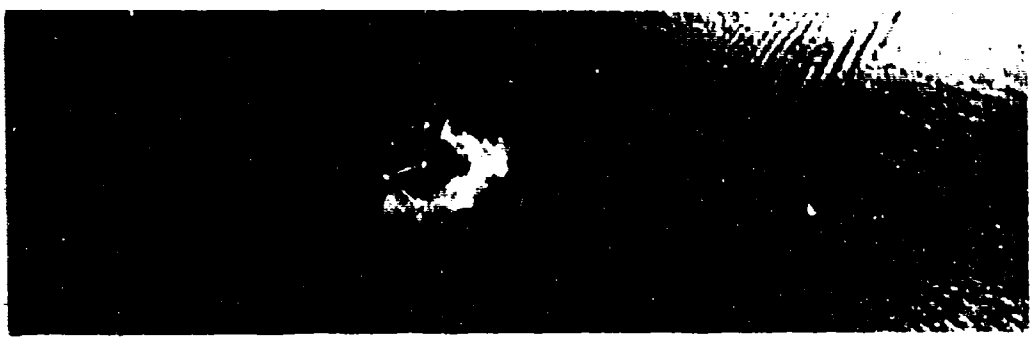
independent evaluations of the equivalent radii by the use of the same raw data reveals that the relative difference in r_e^2 is a few percents.

The early stages of dispersion are difficult to sample since the patch has small areal extent and any significant disturbance of the patch by a sampling vessel must be avoided. An example shows clearly how a small disturbance of unknown cause created a trigger action on the diffusion of patch. Figs. 4a through 4d are serial photographs of a dye patch at early time. It may be seen from the figures that at time between $4^m\ 51^s$ and $6^m\ 47^s$ after release, the leading edge of the patch turned 180° while lagging behind and moving relative to the trianglar drogue with a length of 7.3 meters. The existence of small-scale eddying motion is evident. By $9^m\ 08^s$ after release the leading edge had rejoined the main body of the patch resulting in an elliptical shape. After $11^m\ 40^s$ after release, the mixing of dye had taken place internally so that a fairly large patch of 50 meters in scale appeared (for details see Carter, 1967). A simple calculation based on Joseph and Sendner theory with P = 1 cm/s shows that it would have taken more than an hour for the initial patch to grow into a patch of 50 meters in diameter without the effect of the disturbance.

With a view toward minimizing the disturbances due to a sampling vessel yet still determining the variances of the concentration distribution, Foxworthy et al (1966) carried out dye release experiments in which a small sampling vessel made single crossings of dye patches of small size down to 30 meters. The crossings were aimed so as to lie along the direction of elongation of a patch and the perpendicular direction so that the variances along the principal axes of the concentration distribution could be calculated. The observed concentration distributions have been fitted with a Gaussian curve to compute the variances. The fit was found to be reasonably good especially during the early phase of diffusion.

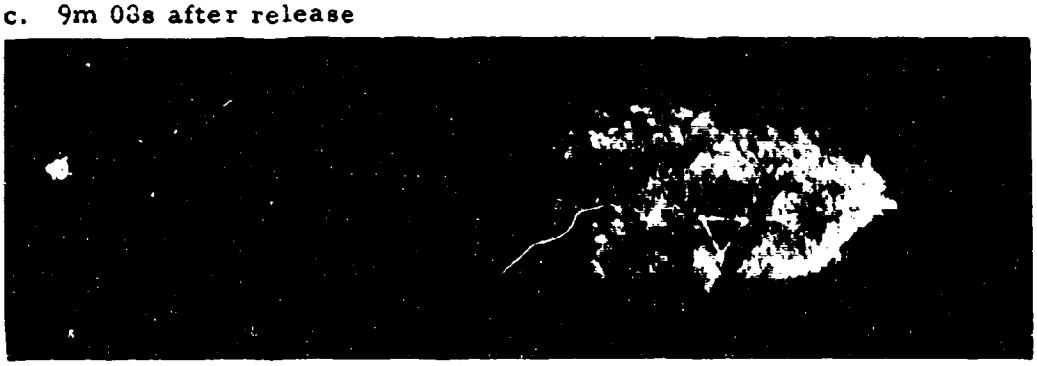


a. 4m 51s after release



b. 6m 47s after release

c. 9m 03s after release



d. 11m 40s after release

Fig. 4 Aerial photographs of dye study in Manokin River

Since the two crossings along the principal axes were not carried out at the same time, an empirical relationship between the estimated variance against diffusion time was determined for each a particular axis. A log-log plot of the variance against time determines a straight line which provides the empirical law expressed as

$$\sigma_x^2 = kt^n \quad ,$$

say for the variance along the major principal axis. It can be seen (Appendix) that, if σ_x^2 and σ_y^2 denote the one-dimensional variances along the principal axes of a two-dimensional normal distribution, the variance σ_{rc}^2 for a radially symmetrical distribution obtained by taking the equivalent radius is given by

$$\sigma_{rc}^2 = 2\,\sigma_x\sigma_y \quad .$$

Actually dye patches elongate in one direction or another, and the distribution of concentration may be represented reasonably well by a two-dimensional Gaussian distribution. Thus, from the empirical laws of σ_x^2 and σ_y^2 with time, we can compute σ_{rc}^2 for specified times of diffusion.

All other dye experiments herein quoted consisted of measuring the horizontal distribution by a proper network of crossings in a dye patch. When the actual pattern of dye distribution is shown in reports (e.g., Joseph, Sendner and Weidemann, 1964), one may compute the mean variance σ_r^2, the individual variances σ_x^2 and σ_y^2, the degree of elongation σ_y/σ_x etc., in addition to the variance for a radially symmetrical distribution.

An apparent diffusivity K_a defined by $\frac{1}{4}\frac{\sigma_{rc}^2}{t}$ and a scale of diffusion defined by $3\ \sigma_{rc}$ are also computed; the scale of

diffusion is arbitrarily taken as the diameter of a radially symmetric patch within which 95% of substance remains (see Appendix). The results of the computations are summarized in Table 1.

4. Diffusion diagrams. By plotting the value of the variance σ_{rc}^2 against the time of diffusion, we obtain a basic diffusion-diagram (Fig. 5). The variance ranges from 10^7 cm^2 up to 10^{13} cm^2, while the diffusion time ranges from 2 hours to nearly a month. For reference purposes, we include the diffusion data in the Manokin River (a narrow tidal tributary to Chesapeake Bay) and the Banana River (s shallow basin). Horizontal diffusion in these regimes should be regarded as two-dimensional. Though the points in the diagram (Fig. 5) scatter somewhat, an obvious trend is recognized that the variance grows much faster in the power of t than a linear increase with time; a Fickian diffusion characterized by a constant diffusivity would result in the variance which grows linearly with diffusion time. Thus horizontal diffusion in the sea is a process in which the apparent diffusivity would increase with the time of diffusion or with the scale of diffusion. A linear fit to all the data points in Fig. 5 gives the relationship that

$$\sigma_{rc}^2 = 0.0108 \cdot t^{2.34} \quad (\sigma_{rc}^2: \text{cm}^2,\ t: \text{sec}) \ . \tag{5}$$

Using diffusion data prior to 1961, Okubo (1962) obtained empirically the similar relationship between σ_{rc}^2 and t:

$$\sigma_{rc}^2 = 0.006 \cdot t^{2.5} \quad (\sigma_{rc}^2: \text{cm}^2,\ t: \text{sec}) \ .$$

Although the quality of the old diffusion data is poor in comparison with that of the new data used in this report, the general

Table 1 Computation of diffusion characteristics

Experiment	t(sec)	σ_{rc}^2 (cm^2)	σ_{rc} (cm)	K_a (cm^2/sec)	ℓ (cm)	ρ	σ_r^2 (cm^2)	Method of Computation of σ_{rc}^2	Source of Data
"RHENO" (1965) North Sea	2.7×10^5	6.1×10^{10}	2.47×10^5	5.65×10^4	7.41×10^5	NA	NA	GD	Weidemann and Sendner (Private communication)
	4.86×10^5	2.05×10^{11}	4.53×10^5	1.05×10^5	1.36×10^6	NA	NA	GD	
	8.1×10^5	4.8×10^{11}	6.93×10^5	1.48×10^5	2.08×10^6	NA	NA	DC	
	9.73×10^5	1.08×10^{12}	1.04×10^6	2.78×10^5	3.12×10^6	NA	NA	DC	
	1.22×10^6	1.50×10^{12}	1.22×10^6	3.08×10^5	3.66×10^6	NA	NA	DC	
	1.8×10^6	3.8×10^{12}	1.95×10^6	5.2×10^5	5.85×10^6	NA	NA	DC	
	2.01×10^6	1.75×10^{13}	4.17×10^6	2.2×10^6	1.24×10^7	NA	NA	GE	
1964-V North Sea	1.69×10^5	1.0×10^{10}	1.0×10^5	1.48×10^4	3.0×10^5	NA	NA	DC	Weidemann and Sendner (Private communication)
	2.63×10^5	2.9×10^{10}	1.70×10^5	2.76×10^4	5.1×10^5	NA	NA	GD	
	4.14×10^5	1.18×10^{11}	3.44×10^5	7.2×10^4	1.03×10^6	NA	NA	GD	
	7.02×10^5	6.3×10^{11}	7.95×10^5	2.2×10^5	2.38×10^6	NA	NA	GE	
	9.0×10^5	1.48×10^{12}	1.2×10^6	4.1×10^5	3.6×10^6	NA	NA	GE	
Oct 1962-III North Sea H=60-65 m	2.38×10^5	1.27×10^{10}	1.13×10^5	1.34×10^4	3.39×10^5	0.25	2.73×10^{10}	GD	Joseph, Sendner, and Weidemann (1964)
	4.1×10^5	2.65×10^{10}	1.63×10^5	1.61×10^4	4.90×10^5	0.18	7.7×10^{10}	GD	

Table 1 Cont'd

Experiment	t(sec)	σ_{rc}^2(cm^2)	σ_{rc}(cm)	K_a(cm^2/sec)	ℓ(cm)	ρ	σ_r^2(cm^2)	Method of Computation of σ_{rc}^2	Source of Data
Sept 1962-II North Sea H=60-65 m	6.8×10^4	1.5×10^9	3.9×10^4	5.5×10^3	1.17×10^5	0.33	2.48×10^9	GD	Joseph, Sendner, and Weidemann (1964)
	9.65×10^4	3.1×10^9	5.6×10^4	8.1×10^3	1.7×10^5	0.30	5.58×10^9	GD	
	1.59×10^5	1.5×10^{10}	1.2×10^5	2.36×10^4	3.7×10^5	0.33	2.48×10^{10}	GD	
	2.47×10^5	2.4×10^{10}	1.55×10^5	2.42×10^4	4.65×10^5	0.10	1.2×10^{11}	GD	
Nov 1961-I North Sea H=40 m	7.6×10^4	2.7×10^9	5.2×10^4	8.9×10^3	1.56×10^5	0.33	4.45×10^9	DC	Joseph, Sendner, and Weidemann (1964)
	1.73×10^5	1.1×10^{10}	1.05×10^5	1.58×10^4	3.15×10^5	0.28	2.09×10^{10}	DC	
	2.49×10^5	5.1×10^{10}	2.26×10^5	5.1×10^4	6.78×10^5	0.20	1.33×10^{11}	GD	
Mar 1962 No. 1 off Cape Kennedy H=6-15 m	1.85×10^5	4.77×10^{10}	2.18×10^5	6.5×10^4	6.54×10^5	0.51	5.92×10^{10}	GD	Carter and Okubo (1965)
Mar 1962 No. 2 off Cape Kennedy H=8-15 m	1.66×10^5	2.22×10^{10}	1.49×10^5	3.3×10^4	4.47×10^5	0.19	6.11×10^{10}	GD	Carter and Okubo (1965)
	2.29×10^5	8.4×10^{10}	2.90×10^5	9.2×10^4	8.7×10^5	0.32	1.43×10^{11}	GD	

Table 1 Cont'd

Experiment	t(sec)	σ_{rc}^2 (cm^2)	σ_{rc} (cm)	K_a (cm^2/sec)	ℓ (cm)	ρ	σ_r^2 (cm^2)	Method of Computation of σ_{rc}^2	Source of Data
Apr 1962 No. 3 off Cape Kennedy H=20 m	7.6×10^4	1.58×10^9	4.0×10^4	5.2×10^3	1.2×10^5	0.36	2.45×10^9	GD	Carter and Okubo (1965)
	2.56×10^5	1.1×10^{10}	1.05×10^5	1.08×10^4	3.15×10^5	0.22	2.64×10^{10}	GD	
Aug 1962 No. 4 off Cape Kennedy H=20-30 m	2.69×10^5	2.42×10^{10}	1.55×10^5	2.3×10^4	4.65×10^5	NA	NA	GD	Carter and Okubo (1965)
	2.84×10^5	1.65×10^{10}	1.29×10^5	1.46×10^4	3.87×10^5	0.4	2.39×10^{10}	GD	
	3.56×10^5	2.82×10^{10}	1.68×10^5	1.98×10^4	5.04×10^5	0.19	7.76×10^{10}	GD	
Aug 1962 No. 5 off Cape Kennedy H=20-30 m	1.97×10^5	8.2×10^{10}	2.86×10^5	1.04×10^5	8.58×10^5	0.3	1.47×10^{11}	GD	Carter and Okubo (1965)
Aug 1962 No. 6 off Cape Kennedy H=20-30 m	1.12×10^5	6.3×10^9	8.0×10^4	1.42×10^4	2.4×10^5	0.14	2.33×10^{10}	GD	Carter and Okubo (1965)
	1.85×10^5	2.8×10^{10}	1.67×10^5	3.8×10^4	5.01×10^5	0.13	1.10×10^{11}	GD	
	2.66×10^5	8.95×10^{10}	2.99×10^5	8.4×10^4	8.97×10^5	0.16	2.91×10^{11}	GD	

Table 1 Cont'd

Experiment	t(sec)	σ_{rc}^2(cm^2)	σ_{rc}(cm)	K_a(cm^2/sec)	ℓ(cm)	ρ	σ_r^2(cm^2)	Method of Computation of σ_{rc}^2	Source of Data
Sept 1962-II North Sea H=60-65 m	6.8×10^4	1.5×10^9	3.9×10^4	5.5×10^3	1.17×10^5	0.33	2.48×10^9	GD	Joseph, Sendner, and Weidemann (1964)
	9.65×10^4	3.1×10^9	5.6×10^4	8.1×10^3	1.7×10^5	0.30	5.58×10^9	GD	
	1.59×10^5	1.5×10^{10}	1.2×10^5	2.36×10^4	3.7×10^5	0.33	2.48×10^{10}	GD	
	2.47×10^5	2.4×10^{10}	1.55×10^5	2.42×10^4	4.65×10^5	0.10	1.2×10^{11}	GD	
Nov 1961-I North Sea H=40 m	7.6×10^4	2.7×10^9	5.2×10^4	8.9×10^3	1.56×10^5	0.33	4.45×10^9	DC	Joseph, Sendner, and Weidemann (1964)
	1.73×10^5	1.1×10^{10}	1.05×10^5	1.58×10^4	3.15×10^5	0.28	2.09×10^{10}	DC	
	2.49×10^5	5.1×10^{10}	2.26×10^5	5.1×10^4	6.78×10^5	0.20	1.33×10^{11}	GD	
Mar 1962 No. 1 off Cape Kennedy H=6-15 m	1.85×10^5	4.77×10^{10}	2.18×10^5	6.5×10^4	6.54×10^5	0.51	5.92×10^{10}	GD	Carter and Okubo (1965)
Mar 1962 No. 2 off Cape Kennedy H=8-15 m	1.66×10^5	2.22×10^{10}	1.49×10^5	3.3×10^4	4.47×10^5	0.19	6.11×10^{10}	GD	Carter and Okubo (1965)
	2.29×10^5	8.4×10^{10}	2.90×10^5	9.2×10^4	8.7×10^5	0.32	1.43×10^{11}	GD	

Table 1 Cont'd

Experiment	t(sec)	σ_{rc}^2 (cm^2)	σ_{rc} (cm)	K_a (cm^2/sec)	ℓ (cm)	ρ	σ_r^2 (cm^2)	Method of Computation of σ_{rc}^2	Source of Data
Apr 1962 No. 3 off Cape Kennedy H=20 m	7.6×10^4	1.58×10^9	4.0×10^4	5.2×10^3	1.2×10^5	0.36	2.45×10^9	GD	Carter and Okubo (1965)
	2.56×10^5	1.1×10^{10}	1.05×10^5	1.08×10^4	3.15×10^5	0.22	2.64×10^{10}	GD	
Aug 1962 No. 4 off Cape Kennedy H=20-30 m	2.69×10^5	2.42×10^{10}	1.55×10^5	2.3×10^4	4.65×10^5	NA	NA	GD	Carter and Okubo (1965)
	2.84×10^5	1.65×10^{10}	1.29×10^5	1.46×10^4	3.87×10^5	0.4	2.39×10^{10}	GD	
	3.56×10^5	2.82×10^{10}	1.68×10^5	1.98×10^4	5.04×10^5	0.19	7.76×10^{10}	GD	
Aug 1962 No. 5 off Cape Kennedy H=20-30 m	1.97×10^5	8.2×10^{10}	2.86×10^5	1.04×10^5	8.58×10^5	0.3	1.47×10^{11}	GD	Carter and Okubo (1965)
Aug 1962 No. 6 off Cape Kennedy H=20-30 m	1.12×10^5	6.3×10^9	8.0×10^4	1.42×10^4	2.4×10^5	0.14	2.33×10^{10}	GD	Carter and Okubo (1965)
	1.85×10^5	2.8×10^{10}	1.67×10^5	3.8×10^4	5.01×10^5	0.13	1.10×10^{11}	GD	
	2.66×10^5	8.95×10^{10}	2.99×10^5	8.4×10^4	8.97×10^5	0.16	2.91×10^{11}	GD	

Table 1 Cont'd

Experiment	t(sec)	σ_{rc}^2(cm^2)	σ_{rc}(cm)	K_a(cm^2/sec)	ℓ(cm)	ρ	σ_r^2(cm^2)	Method of Computation of σ_{rc}^2	Source of Data
Apr 1964 Banana River H=2 m	1.4×10^4	6.25×10^7	7.93×10^3	1.12×10^3	2.38×10^4	0.8	6.4×10^7	GD	Carter and Okubo (1965)
	2.4×10^4	2.62×10^8	1.62×10^4	2.75×10^3	4.86×10^4	0.5	3.28×10^8	GD	
	8.8×10^4	3.1×10^8	1.76×10^4	8.8×10^2	5.28×10^4	0.3	5.62×10^8	GD	
	1.1×10^5	3.35×10^8	1.83×10^4	7.7×10^2	5.5×10^4	0.26	6.87×10^8	GD	
	1.97×10^5	2.01×10^9	4.48×10^4	2.6×10^3	1.34×10^5	0.13	8.04×10^9	GD	
Jun 1967 Manokin River	1.7×10^5	4.25×10^9	6.5×10^4					GE	Carter (1967)
	1.9×10^5	6.58×10^9	8.1×10^4					GE	
	2.6×10^5	1.16×10^{10}	1.08×10^5					GE	
Oct 1961 New York Bight H=30 m	3.38×10^4	1.42×10^9	3.77×10^4	1.05×10^4	1.13×10^5	0.14	5.25×10^9	GD	Costin, Davis, Gerard, and Katz (1963)
	4.13×10^4	1.38×10^9	3.72×10^4	8.4×10^3	1.11×10^5	NA	NA	GD	
	4.45×10^4	1.65×10^9	4.06×10^4	9.3×10^3	1.22×10^5	0.13	6.52×10^9	GD	
	7.9×10^4	1.29×10^{10}	1.13×10^5	4.1×10^4	3.39×10^5	NA	NA	GD	
	9.13×10^4	1.84×10^{10}	1.35×10^5	5.04×10^4	4.05×10^5	0.25	3.95×10^{10}	GE	
Oct 1962 No. a off Southern California H=15 m	6×10^3	1.23×10^7	3.51×10^3	5.13×10^2	1.05×10^4	0.17	3.64×10^7	GD	Foxworthy, Tibby, and Barsom (1966)
	1.2×10^4	2.42×10^7	4.92×10^3	5.04×10^2	1.48×10^4	0.16	7.63×10^7	GD	

Table 1 Cont'd

Experiment	t(sec)	σ_{rc}^2(cm^2)	σ_{rc}(cm)	K_a(cm^2/sec)	ℓ(cm)	ρ	σ_r^2(cm^2)	Method of Computation of σ_{rc}^2	Source of Data
Nov 1962 No. b off Southern California H=15 m	6 x10^3	2.58x10^7	5.08x10^3	1.07x10^3	1.52x10^4	0.40	3.8 x10^7	GD	Foxworthy, Tibby, and Barsom (1966)
	1.2 x10^4	5.57x10^7	7.46x10^3	1.16x10^3	2.26x10^4	0.32	9.64x10^7	GD	
Mar 1963 No. c off Southern California H=15 m	6 x10^3	6.44x10^7	8.02x10^3	2.68x10^3	2.41x10^4	0.27	1.27x10^8	GD	Foxworthy, Tibby, and Barsom (1966)
	1.2 x10^4	2.31x10^8	1.52x10^4	4.81x10^3	4.56x10^4	0.25	4.76x10^8	GD	
Apr 1963 No. d off Southern California H=15 m	6 x10^3	1.60x10^7	4.00x10^3	6.67x10^2	1.2 x10^4	0.29	5.04x10^7	GD	Foxworthy, Tibby, and Barsom (1966)
	1.2 x10^4	3.46x10^7	5.88x10^3	7.21x10^2	1.76x10^4	0.21	8.78x10^7	GD	
Aug 1963 No. e off Southern California H=15 m	6 x10^3	4.56x10^7	6.75x10^3	1.9 x10^3	2.02x10^4	0.26	9.22x10^7	GD	Foxworthy, Tibby, and Barsom (1966)
	1.2 x10^4	1.41x10^8	1.19x10^4	2.95x10^3	3.57x10^4	0.21	3.49x10^8	GD	
	1.8 x10^4	3.6 x10^8	1.90x10^4	5.0 x10^3	5.70x10^4	0.18	7.66x10^8	GD	

Table 1 Cont'd

Experiment	t(sec)	σ_{rc}^2(cm^2)	σ_{rc}(cm)	K_a(cm^2/sec)	ℓ(cm)	ρ	σ_r^2(cm^2)	Method of Computation of σ_{rc}^2	Source of Data
Dec 1963	6 x10^3	2.25x10^7	4.74x10^3	9.37x10^2	1.42x10^4	0.40	3.28x10^7	GD	Foxworthy, Tibby and Barsom (1966)
No. f off Southern California H=15 m	1.2 x10^4	5.58x10^7	7.47x10^7	1.16x10^3	2.24x10^4	0.30	1.01x10^8	GD	

H: depth of water

t: Diffusion time

σ_{rc}^2: Horizontal variance for radially symmetrical distribution

$K_a \equiv \frac{1}{4}\frac{\sigma_{rc}^2}{t}$: apparent diffusivity

$\ell \equiv 3\,\sigma_{rc}$: Scale of diffusion

$\rho \equiv \frac{\sigma_Y}{\sigma_X}$: Degree of elongation

σ_r^2 : Mean variance

NA: Not available

(Method of computation of σ_{rc}^2)

GD: Fit to Gaussian distribution

GE: Fit to generalized exponential distribution

DC: Direct computation

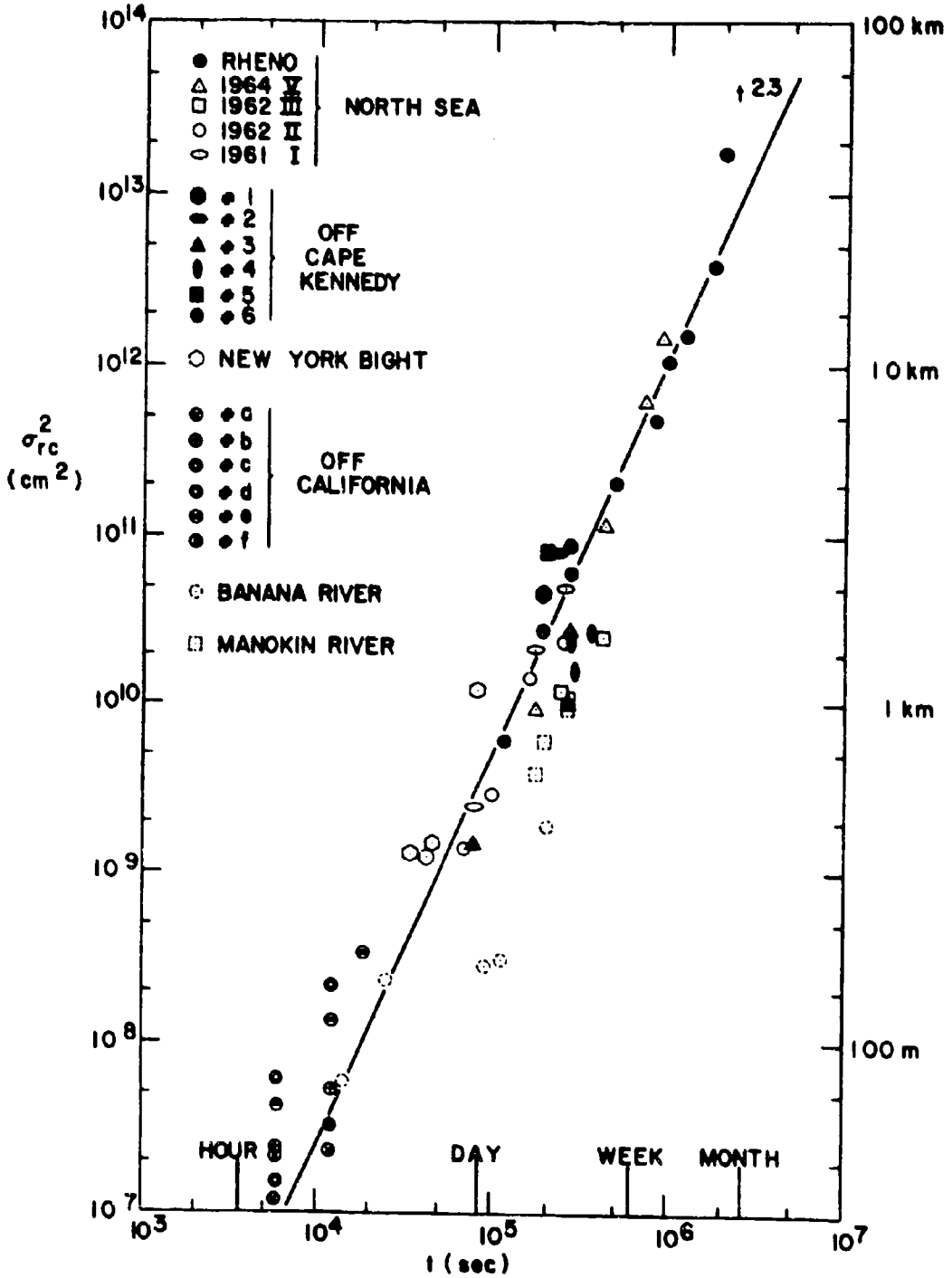


Fig. 5. Variance, σ_{rc}^2, vs. diffusion time. (new data)

conclusion remains unchanged. Although the new data may be fitted locally in time to the power 3.0, the overall slope of the log σ_{ro}^2-log t line is noticeably smaller than 3.0; the third power of t for the variance has been predicted by the similarity theory of turbulence (see Section 5).

In Fig. 6 we plot all the data, old and new together, in one diagram. It should be noticed that the observed variance at a diffusion time varies as much as an order of magnitude. The variability in the variance seems to be more noticeable at smaller diffusion times. For large times of diffusion, a patch of diffusing substance itself takes average conditions; a variety of wind and current situations is encountered and the average effect does not differ much from one experiment to another.

Fig. 7 shows the relationship between the apparent diffusivity and the scale of diffusion. A linear fit to the new data points gives the relationship that

$$K_a = 0.0103 \cdot \ell^{1.15} \qquad (K_a: \text{cm}^2/\text{s},\ \ell: \text{cm}) \ . \tag{6}$$

The exponent of ℓ is definitely smaller than 4/3; the 4/3 law of oceanic diffusion has been proposed by several investigators (see Bowden, 1962), and explained by them with the similarity theory of turbulence. So far as dye diffusion experiments are concerned, however, the apparent diffusivity does not obey the 4/3 law, although there remains the possibility that the 4/3 law may be applied locally in scale to the data. Okubo (1962) came to the same conclusion by the use of the old data; the old relationship was given by

$$K_r \propto \ell^{1.19} \ ,$$

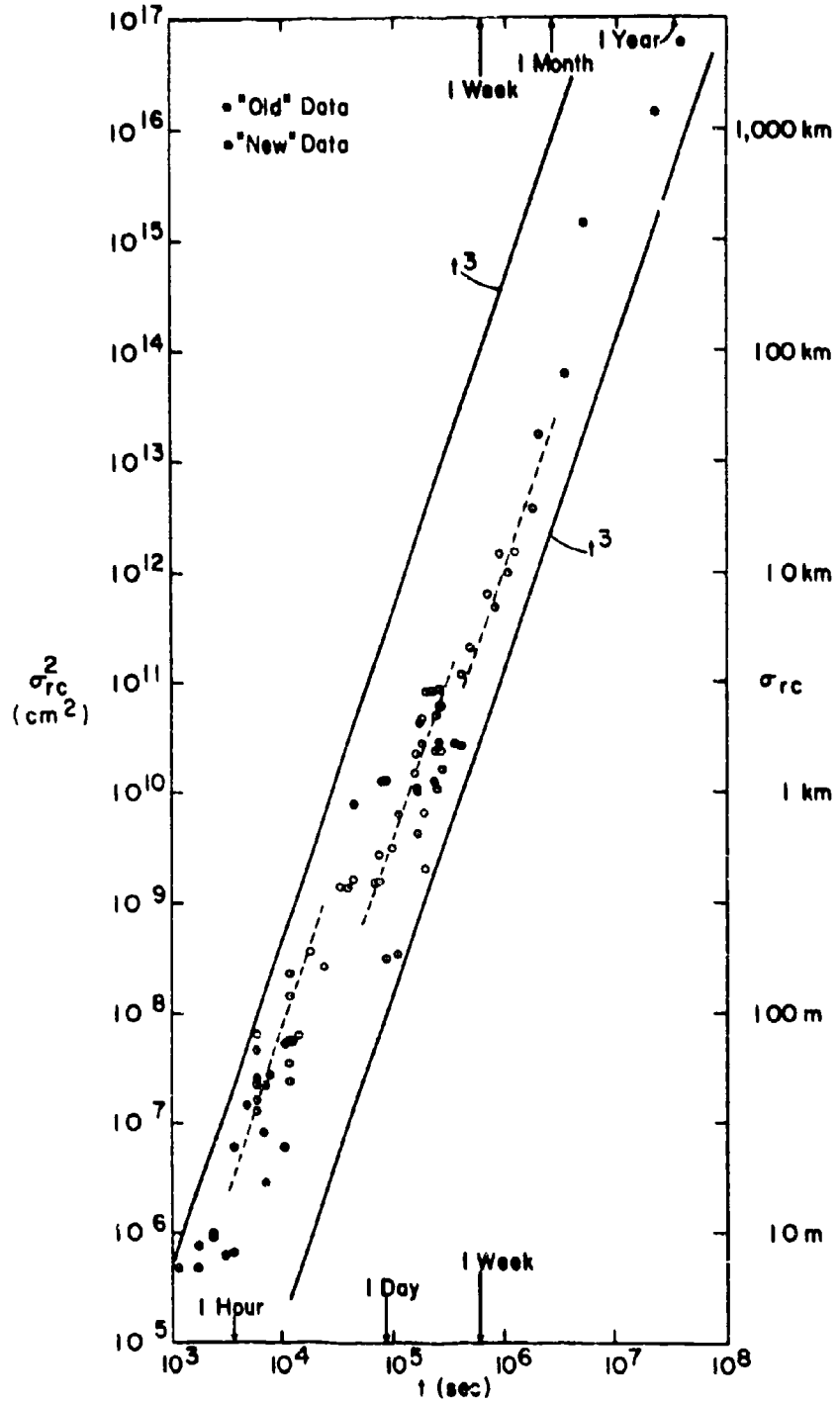


Fig. 6. Variance vs. diffusion time (old and new data): fit of the t^3 relation locally.

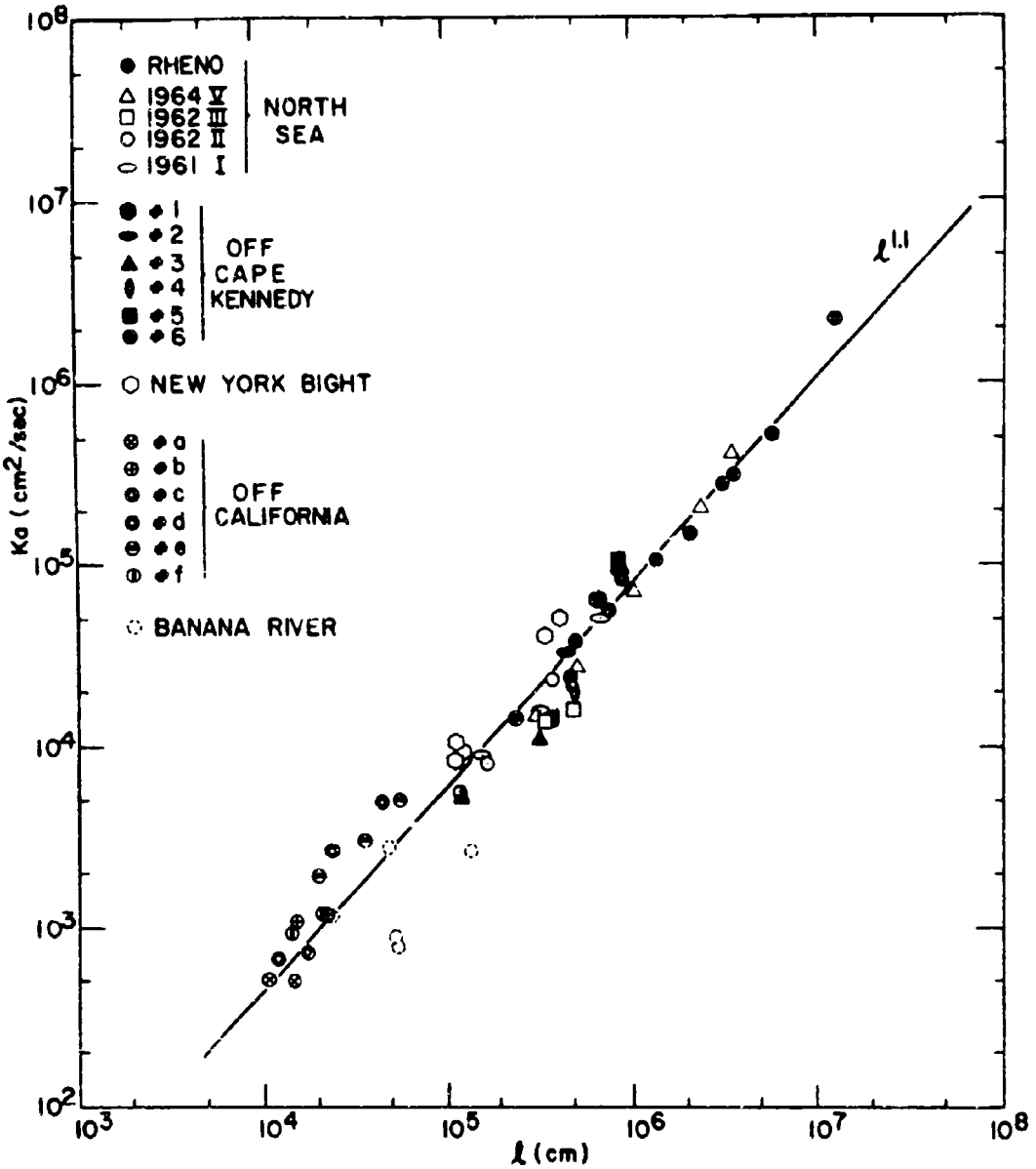


Fig. 7. Apparent diffusivity, K_a, vs. scale of diffusion, $\ell \equiv 3\ \sigma_{rc}$ (new data).

where K_r was defined by $\frac{1}{2}\frac{d\sigma_{rc}^2}{dt}$ and ℓ by $4\,\sigma_{rc}$.

In Fig. 8 we plot all the data, old and new, together in one diagram of the K_a - ℓ relationship. The diagrams (Figs. 5 and 7) can be used for an empirical approach to predict the horizontal spread of substance from an instantaneous, small source, within a certain range of variability in the characteristics. An example of predictions by the use of the relations (5) and (6) is given in Table 2.

5. Discussion. The new diffusion diagrams herein constructed using the data of recent dye experiments provide not only a practical means of predicting the characteristics of oceanic diffusion but also a certain clue concerning the spectrum of oceanic turbulence. As for the prediction of horizontal diffusion from an instantaneous point source, Figs. 5 and 7 can be used as a general guide. Irrespective of the detailed oceanographic conditions, the horizontal diffusion exhibits a general trend; the variance increases with time at a power between 2 and 3. It is, however, to be remembered that the variance may vary, probably depending on the oceanographical conditions, by an order of magnitude or more for the same diffusion time. Only a limited number of dye studies have been made in which the horizontal spread of substance is compared with the environmental factors, such as the stability of water column (Foxworthy et al, 1966, Kullenberg, 1967).

Foxworthy et al (1966) examined the effect of water column stability and wind speed on the variances in the directions of the principal axes of diffusing patch. At any given average value of wind speed ranging from 2 to 14 kt, lower values of dilution (a measure of the rate of dispersion) are associated with higher values of average stability. This effect is controlled primarily by the suppression of the vertical diffusion by increasing stability. The effect of stability

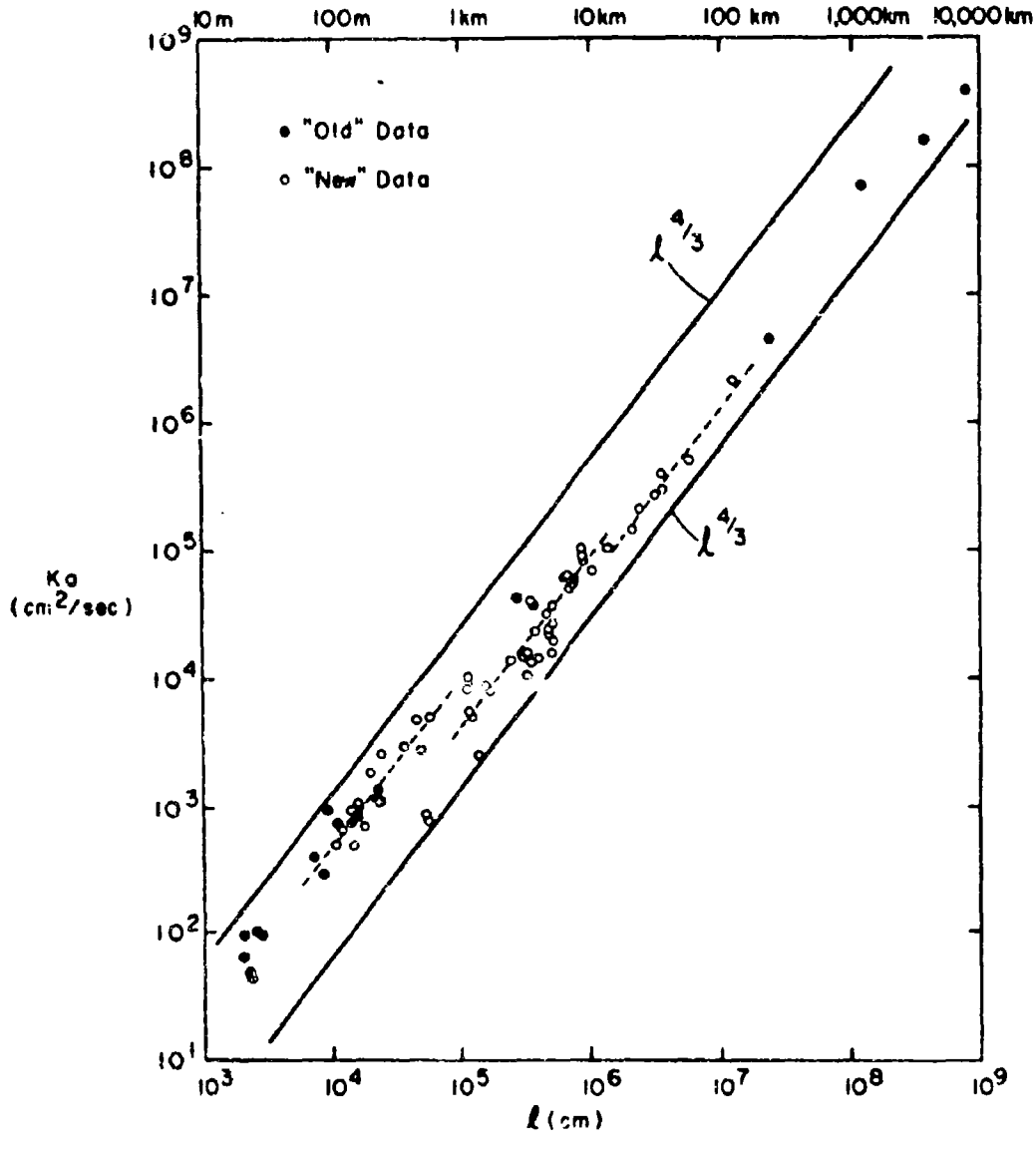


Fig. 8. Apparent diffusivity vs. scale of diffusion (old and new data): fit of the $\ell^{4/3}$ law locally.

Table 2 An example of predictions of diffusion characteristics

a) Variance and standard deviation of a radially symmetric patch vs diffusion time

t(sec)		σ_{rc}^2(cm^2)	σ_{rc}(cm)
3.6×10^3	(1 hour)	2.3×10^6	1.5×10^3
10^4		2.5×10^7	5.0×10^3
3.6×10^4	(10 hours)	4.8×10^8	2.2×10^4
8.64×10^4	(1 day)	3.6×10^9	6.0×10^4
10^5		5.4×10^9	7.4×10^4
6.05×10^5	(1 week)	3.6×10^{11}	6.0×10^5
10^6		1.2×10^{12}	1.1×10^6
2.59×10^6	(1 month)	1.2×10^{13}	3.3×10^6

b) Apparent diffusivity vs scale of diffusion

ℓ(cm)	K_a(cm^2/sec)
10^4	4.1×10^2
5×10^4	2.6×10^3
10^5	5.8×10^3
5×10^5	3.7×10^4
10^6	8.2×10^4
5×10^6	5.3×10^5
10^7	1.2×10^6

on horizontal dispersion is inconclusive, however. Thus, in the range of 2 to 8 kt wind, an increase in σ_X^2 with decreasing stability is suggested, but there are insufficient data on which to base a firm conclusion. Similar stability appears to have little or no definite effect on σ_Y^2. Although the data scatter considerably on a plot of the variances and wind speed, there is indicated a trend of increasing σ_X^2 with increasing wind speed, whereas there appears to be no definite correlation between σ_Y^2 and wind speed. The wind speed of our concern may be regarded as a measure of a characteristic current speed in the water column within which diffusion takes place.

It has recently become apparent that velocity shear can play an important role in the horizontal spread of contaminants in the sea. The basic result of the effect of current shear is that an effective longitudinal dispersion is produced by the combination of the gradient of mean velocity and turbulent mixing in the same direction. In the case of a uniform vertical shear in the horizontal current, the longitudinal and lateral variances, σ_X^2 and σ_Y^2, can be expressed as (Carter and Okubo, 1965):

$$\sigma_X^2 = \frac{1}{6}\,\Omega_z^2 A_z\, t^3 \quad , \qquad (7)$$

$$\sigma_Y^2 = 2\,A_y t \quad ,$$

where Ω_z : the constant vertical shear, A_z: the vertical eddy diffusivity, Ay: the lateral eddy diffusivity.

It may be speculated that the lateral eddy diffusivity would be relatively unaffected by stability. That is, σ_Y^2 would be relatively uncorrelated to stability. On the other hand, the tendency of increasing σ_X^2 with increasing wind speed, i.e., increasing shear, is evident from the expression (7), and the decrease of σ_X^2 with increasing stability can also be seen because of the suppression

of the vertical eddy diffusivity by increasing stability. Thus the simple shear-diffusion model can interpret, at least qualitatively, the experimental results discovered by Foxworthy et al. The shear effect on horizontal diffusion implies that the variance for a radially symmetrical distribution is closely related to the current shear and stability aside from the time dependence. The variability of the variance at a fixed diffusion time is thus nothing but a reflection of the change in the environmental conditions. A future study should include these environmental factors as parameters in the discussion of the diffusion diagram.

As mentioned previously, a radially symmetrical distribution may not represent properly the actual pattern of oceanic diffusion. In addition, the value of the horizontal variance is always smaller than it should be when we convert the actual distribution of concentration $S(x, y, t)$, into a radially symmetrical one characterized by an equivalent radius (see Appendix). In other words, we underestimate the horizontal variance by an arbitrary amount when taking a radially symmetrical distribution. The correction factor σ_r^2/σ_{rc}^2 depends on the degree of asymmetry in the actual distribution (Appendix). The more elongated the actual patch of substance, the larger the correction factor should be.

We have observed quite often that the patch tends to elongate more and more as time goes on (see a paper by Joseph, Sendner and Weidemann, 1964). Thus the correction factor would grow as the diffusion time grows. This raises an important question; "How would σ_r^2 behave with time if we plot σ_r^2, instead of σ_{rc}^2, against t?" At present not much can be answered because some of the large scale diffusion data provide information only on the equivalent radius. Fig. 9 shows a plot of σ_r^2 vs. t. It suggests more clearly than Fig. 5 that the t^3 law of σ_r^2 should be valid locally in time.

The turbulent diffusion of a patch of dye is a problem in relative

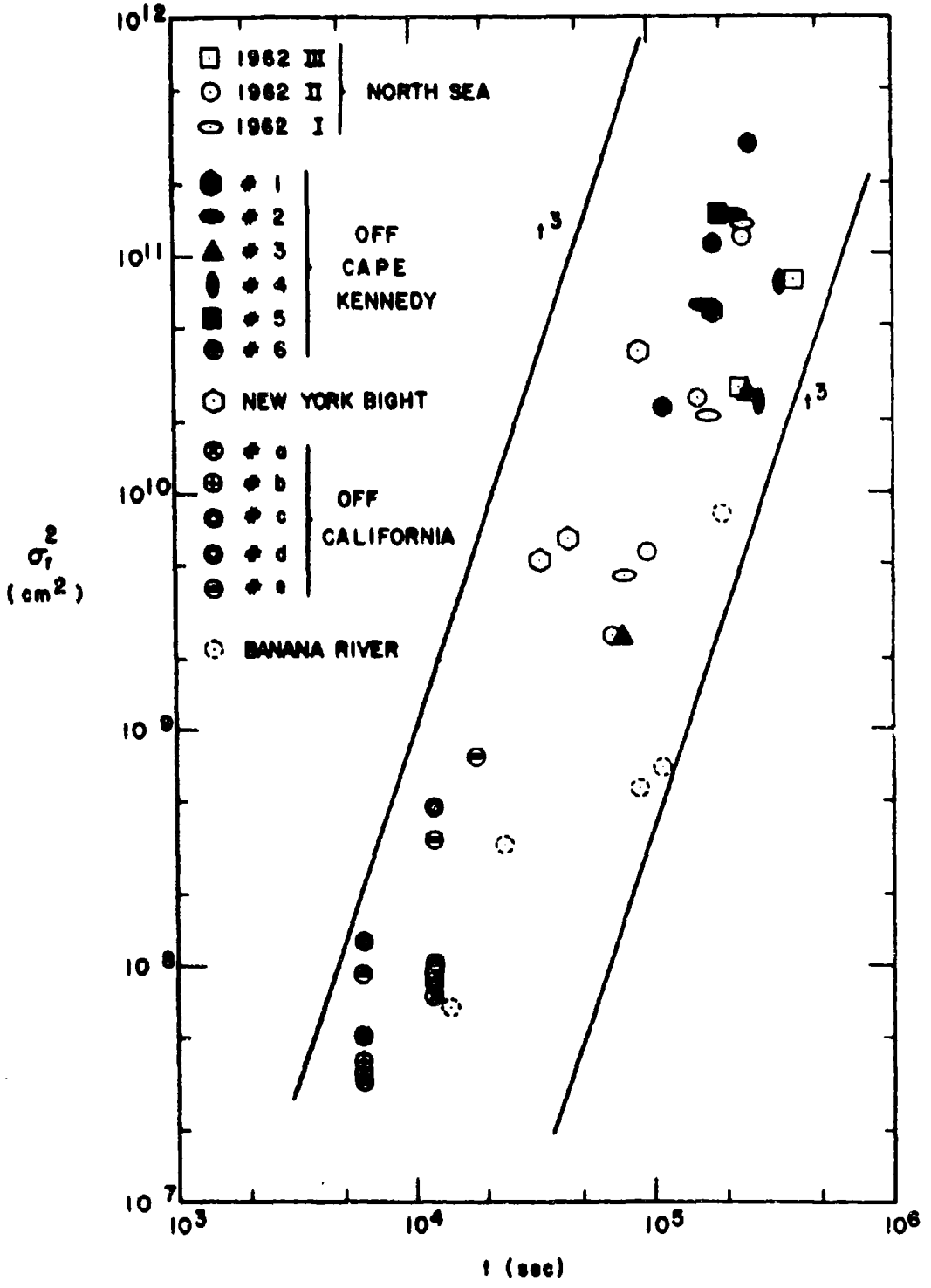


Fig. 9. Mean variance, σ_r^2, vs. diffusion time.

diffusion. A theoretical treatment of this problem has been given by Batchelor (1952). With the aid of dimensional arguments, Batchelor finds a regime of relative diffusion if the initial size of the patch is infinitesimal. This regime is characterized by the following

$$\sigma_r^2 = C E t^3 \quad , \tag{8}$$

where C is a constant of order unity and E is the rate of energy dissipation per unit mass through turbulence. It is easy to derive from (8) that the apparent diffusivity grows as the 4/3 power of patch size:

$$K_a \propto E^{1/3} \sigma_r^{4/3} \quad . \tag{9}$$

This is Richardson's (1926) well-known law of relative diffusion, the physical meaning of which is that relative diffusion is an accelerating process, the rate of growth increasing with the size of the patch.

The basic concept involved in the relationship (8) and also in the Richardson's law is that the eddies responsible for the horizontal spread of substance lie in the "inertial subrange" (Kolmogorov, 1941). These eddies receive their energy from the larger eddies and pass it to eddies smaller than themselves. No direct energy is supplied to those eddies from external sources and the energy dissipation due to viscosity is not significant. Their properties thus depend only on the rate of energy transfer which must be equal to the rate of energy dissipation if the energy of the eddies remains stationary.

Under natural conditions, however, there is often the possibility of energy being added directly to such eddies by wind systems, tidal currents, or, on a still smaller scale, by waves (Stommel, 1949, Ozmidov, 1965). These excited eddies could not then be strictly in the inertial subrange, so that results derived from that concept would not necessarily be applicable. Ozmidov (1965),

nevertheless, suggests the existence of locally inertial subranges separated by two scales of eddies where an influx of external energy takes place. Fig. 10 shows schematically the spectrum conjectured by Ozmidov. This, in turn, suggests that the law (8) or the Richardson's law may be used locally in time or scale with a different value of the rate of turbulence energy transfer. As the scale of diffusion increases, the smaller the value of this parameter one should expect, since the local supplies of energy tend to be transferred, through the nonlinear interactions, from the larger eddies to the smaller eddies (for some exceptional cases, see Webster 1961). As a consequence, the overall rate of growth of the variance with time should be slower than the cube power law and the overall growth rate of the apparent diffusivity with the scale should be smaller than the 4/3 law.

On the basis of this concept, we fit the law (8) locally to the data points as shown in Fig. 6, from which the values of the rate of energy transfer in the local regimes are computed, assuming the numerical constant C = 1. Similar fit to the data of K_a vs. ℓ is shown in Fig. 8. The result of computation of E is summarized by

$E = 9.7 \cdot 10^{-5}$ cm^2/sec^3 for $2 \cdot 10^4 > t > 4 \cdot 10^3$ sec or $10^5 > \ell > 10^4$ cm

$E = 3.3 \cdot 10^{-6}$ cm^2/sec^3 $3 \cdot 10^5 > t > 5 \cdot 10^4$ sec or $10^6 > \ell > 10^5$ cm

$E = 1 \cdot 10^{-6}$ cm^2/sec^3 $3 \cdot 10^6 > t > 4 \cdot 10^5$ sec or $2 \times 10^7 > \ell > 10^6$ cm

A clear jump in the value of E occurs at a time of a half day to a day. This suggests a local energy supply by tidal motions, amounting to 9.4×10^{-5} cm^2/sec^3 per unit mass of sea water. According to Jeffrey (1952) the average rate of dissipation of tidal energy is estimated at $1.4 \times$ [illegible] ^{19} ergs/sec. If this were distributed uniformly throughout the ocean body, the average rate of dissipation

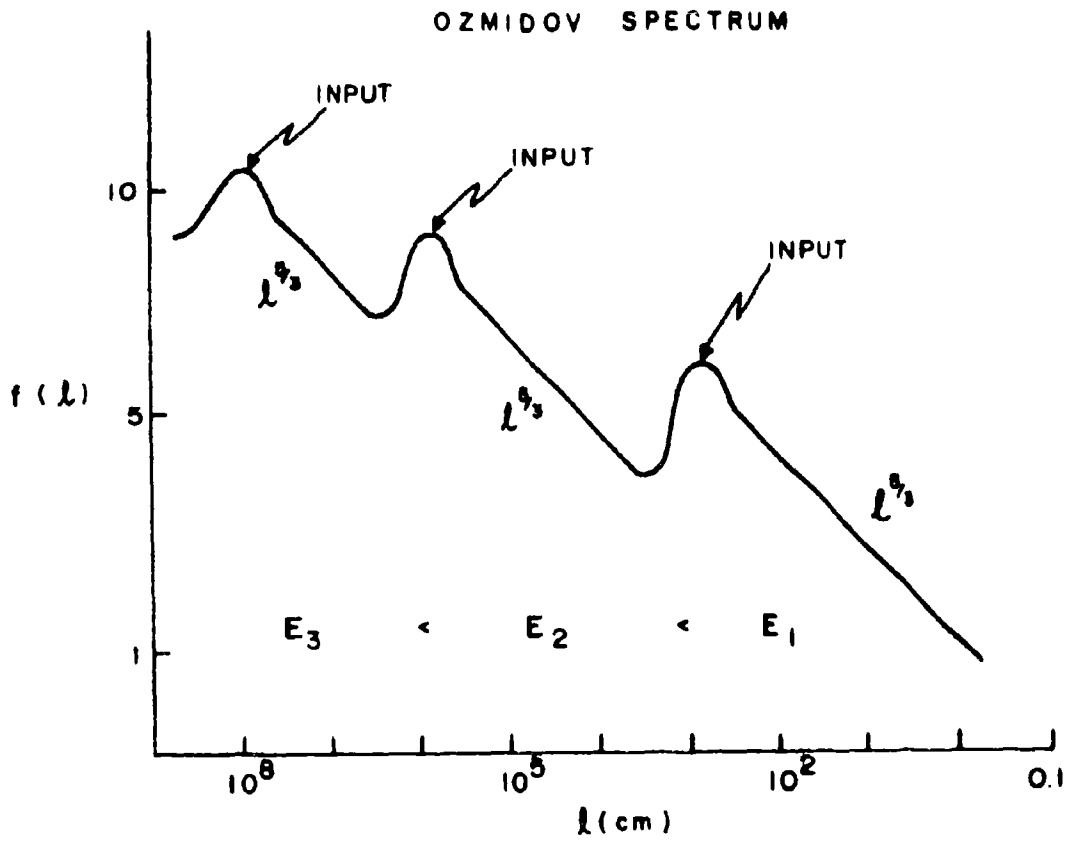


Fig. 10. Ozmidov spectrum for oceanic turbulence.

of tidal energy per unit mass of water would be 1×10^{-5} cm^2/sec^3, which is an order of magnitude smaller than the value of 9.4×10^{-5} cm^2/sec^3. However, it is believed (Knauss, 1956) that the most of the tidal energy is dissipated in shallow seas; thus the local energy supply by tidal motions would be much higher than 10^{-5} cm^2/sec^3 in shallow waters.

There appears to exist another jump in E at a time scale of a few to several days. This is, however, inconclusive not only because of the fact that the data points for the large-scale diffusion have not been corrected to the mean variance, σ_r^2, but also because the quality of those points which cover a scale of more than 100 km is found to be very poor.

6. Final Remarks.

1) The diffusion diagrams herein presented are made exclusively from the data of diffusion experiments in the surface layer of the sea. Very little has been known about the horizontal diffusion in deep water or in the thermocline simply because of the technical difficulties in carrying out dye release experiments there. Doubtless those experiments would provide important clues to our understanding of large-scale mixing in the sea.

2) In order to provide adequate information on the study of oceanic diffusion, a particular manner of data reporting must be used. A good example of data presentation is seen in papers by Joseph, Sendner and Weidemann (1964) and by Carter and Okubo (1965). The following information should be considered as "necessary", whereas some of them have often been missing in reports;

i) Vertical distribution of concentration as a measure of depth of mixing,

ii) Velocity field, especially current shears in the scale of diffusion; a minimum requirement is the mean velocity of the

center of mass of patch during the interval of successive observations,

iii) Initial conditions of release: the duration of release, the depth of release, the initial size of patch, etc.;

iv) Shape information: the degree of elongation, the direction of elongation with respect to the mean flow, etc.; the pattern of the horizontal distribution of concentration should be shown whenever available,

v) Background oceanographic conditions: the stability of water, etc.,

vi) Wind data, sea state, etc.

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APPENDIX

Characteristics of dispersion associated with a two-dimensional distribution

Consider a two-dimensional distribution of concentration from an instantaneous release of a unit amount,

$$S = S(t,x,y): \int_{-\infty}^{\infty}\int Sdxdy = 1 \quad \text{for all } t \geq 0 \quad . \tag{A-1}$$

Thus the mean value of any property $f(x, y, t)$ associated with S is defined by

$$\overline{f} \equiv \int_{-\infty}^{\infty}\int fSdxdy \quad . \tag{A-2}$$

The origin of the orthogonal coordinate system (x, y) with an arbitrary orientation is taken at the center of mass of the distribution S, so that $\overline{x} = \overline{y} = 0$. The mean square distance from the center of mass, i.e., the variance, defining by

$$\sigma_r^2 \equiv \overline{r^2} = \int_{-\infty}^{\infty}\int (x^2 + y^2)\, Sdxdy \quad , \tag{A-3}$$

thus can be expressed as the sum of the mean square distances from the center of mass along the two orthogonal coordinates;

$$\sigma_r^2 = \sigma_x^2 + \sigma_y^2 \quad . \tag{A-4}$$

Let us compute the mean square separations between a pair of substance particles in the patch, $\overline{\ell^2}$:

$$\overline{\ell^2} = \iint |\underset{\sim}{r}_2 - \underset{\sim}{r}_1|^2 \, F(t,\underset{\sim}{r}_1,\underset{\sim}{r}_2) \, d\underset{\sim}{r}_1 \, d\underset{\sim}{r}_2 \quad , \tag{A-5}$$

where $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$ are the position vectors from the origin to two particles, respectively, $\underset{\sim}{\ell} = \underset{\sim}{r}_2 - \underset{\sim}{r}_1$, and $F(t,\underset{\sim}{r}_1,\underset{\sim}{r}_2)$ is the joint probability density function for the two particles (see Fig. A). The integral (A-5) extends over the whole space of $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$.

Since we can take two particles each independently of the other, the joint probability is expressed as the product of the density functions for each particle, that is

$$F(t,\underset{\sim}{r}_1,\underset{\sim}{r}_2) = S(t,\underset{\sim}{r}_1) \, S(t,\underset{\sim}{r}_2) \quad . \tag{A-6}$$

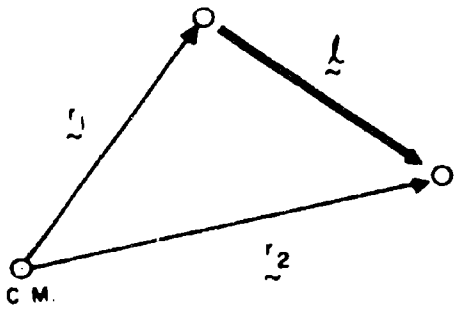
Substitution (A-6) into (A-5) and calculating, we obtain

$$\overline{\ell^2} = \int \underset{\sim}{r}_2^{\,2} \, S(t,\underset{\sim}{r}_2) d\underset{\sim}{r}_2 \int S(t,\underset{\sim}{r}_1) d\underset{\sim}{r}_1$$

$$- 2 \iint \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 \, S(t,\underset{\sim}{r}_1) \, S(t,\underset{\sim}{r}_2) \, d\underset{\sim}{r}_1 \, d\underset{\sim}{r}_2$$

$$+ \int \underset{\sim}{r}_1^{\,2} \, S(t,\underset{\sim}{r}_1) \, d\underset{\sim}{r}_1 \int S(t,\underset{\sim}{r}_2) \, d\underset{\sim}{r}_2 = \overline{\underset{\sim}{r}_2^{\,2}} + \overline{\underset{\sim}{r}_1^{\,2}} - 2\overline{\underset{\sim}{r}_1} \cdot \overline{\underset{\sim}{r}_2} \tag{A-7}$$

$$= \overline{\underset{\sim}{r}_2^{\,2}} + \overline{\underset{\sim}{r}_1^{\,2}} \text{ (since } \overline{\underset{\sim}{r}_1} = \overline{\underset{\sim}{r}_2} = 0) = \overline{r^2} + \overline{r^2} = 2\,\sigma_r^{\,2} \quad .$$



$$\underset{\sim}{l} = \underset{\sim}{r}_2 - \underset{\sim}{r}_1$$

Fig. A. Position and separation vectors.

We thus conclude that the mean square separations between a pair of particles in the patch is equal to the twice of the mean square distances of a particle from the center of mass of the patch.

We shall now specify the distribution by a two-dimensional Gaussian distribution:

$$S(t,x,y) = \frac{1}{2\pi\sigma_1(t)\sigma_2(t)(1-\lambda^2(t))^2}\, e^{-\frac{1}{1-\lambda^2}\left[\frac{x^2}{2\sigma_1^2} - \frac{\lambda}{\sigma_1\sigma_2}xy + \frac{y^2}{2\sigma_2^2}\right]}, \qquad \text{(A-8)}$$

where $\sigma_1^2(t)$ and $\sigma_2^2(t)$ are the variances in the directions of x and y, respectively, and λ is a coefficient of correlation:

$$\lambda \equiv \frac{\overline{xy}}{\sigma_1\sigma_2} . \qquad \text{(A-9)}$$

Equation (A-8) describes that the concentration contours are a set of ellipses with respect to the center of mass.

Rotating the coordinate system (x, y) by a certain angle, we can express S in terms of the principal axes (X, Y).

$$S(t,X,Y) = \frac{1}{2\pi\sigma_X\sigma_Y}\, e^{-\left(\frac{X^2}{2\sigma_X^2} + \frac{Y^2}{2\sigma_Y^2}\right)}, \qquad \text{(A-10)}$$

where σ_X^2 and σ_Y^2 are the variances in the major and minor principal axes, respectively. Obviously,

$$\sigma_r^2 = \sigma_X^2 + \sigma_Y^2 . \qquad \text{(A-11)}$$

We now convert the asymmetric distribution (A-10) into a radially symmetric one by taking the equivalent radius, r_e. Since the area enclosed by a concentration must be equal for both distributions, we have

$$r_e^2 = \left(\frac{\sigma_Y^2}{\sigma_X \sigma_Y}\right) X^2 + \left(\frac{\sigma_X^2}{\sigma_X \sigma_Y}\right) Y^2$$

$$= \left(\frac{X^2}{2\sigma_X^2} + \frac{Y^2}{2\sigma_Y^2}\right) 2\sigma_X \sigma_Y \quad .$$

Hence, (A-10) is reduced to

$$S(t, r_e) = \frac{1}{2\pi\sigma_X\sigma_Y} e^{-\frac{r_e^2}{2\sigma_X\sigma_Y}} \quad . \qquad \text{(A-12)}$$

Define the variance for the radially symmetric distribution by

$$\sigma_{rc}^2 \equiv \int_0^\infty r_e^2 \, S(t, r_e) \, 2\pi r_e \, dr_e \quad .$$

We then obtain

$$\sigma_{rc}^2 = 2 \, \sigma_X \sigma_Y \quad . \qquad \text{(A-13)}$$

Therefore,

$$S(t, r_e) = \frac{1}{\pi\sigma_{rc}^2(t)} e^{-\frac{r_e^2}{\sigma_{rc}^2(t)}} \qquad \text{(A-14)}$$

In other words, the variance for the radially symmetric distribution is equal to the twice of the geometric average of the one-dimensional variances in the principal axes for the original two-dimensional Gaussian distribution.

It should be noted that, in general,

$$\sigma_{rc}^2 \neq 2\,\sigma_x\sigma_y \quad , \tag{A-15}$$

where x and y are orthogonal coordinates in an arbitrary orientation with respect to an elliptical patch of substance.

It is easily seen that the variance for an elliptical distribution never amounts to less than that of the corresponding radially symmetrical distribution, that is

$$\sigma_r^2 - \sigma_{rc}^2 = \sigma_X^2 + \sigma_Y^2 - 2\sigma_X\sigma_Y = (\sigma_X - \sigma_Y)^2 \geq 0 \quad .$$

Define the degree of elongation, ρ , by

$$\rho \equiv \sigma_Y/\sigma_X \quad . \tag{A-16}$$

Then, eliminating σ_X and σ_Y among (A-11), (A-13) and (A-16), we obtain

$$\frac{\sigma_r^2}{\sigma_{rc}^2} = 1 + \frac{(1-\rho)^2}{2\rho} \quad . \tag{A-17}$$

Finally we shall calculate the relative amount of substance contained between r = 0 and r = n σ_{rc} (n > 0) for a radially symmetric distribution of a Gaussian type.

$$\gamma \equiv \int_0^{n\sigma_{rc}} S(t,r_e)\, 2\pi\, r_e\, dr_e$$

$$= \int_0^{n\sigma_{rc}} \frac{1}{2\pi\sigma_{rc}^2}\, e^{-\frac{r_e^2}{2\sigma_{rc}^2}}\, 2\pi\, r_e\, dr_e = \int_0^{n^2} e^{-\eta}\, d\eta = 1 - e^{-n^2} \quad .$$

We thus obtain

$n = 1, \quad \ell = 2\ \sigma_{rc}\ ; \quad \gamma = 0.632,$

$n = 1.5, \quad \ell = 3\ \sigma_{rc}\ ; \quad \gamma = 0.950,$

$n = 2, \quad \ell = 4\ \sigma_{rc}\ ; \quad \gamma = 0.982.$

That is, 95% of substance remains with a diameter of 3 σ_{rc} for all t.

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13 ABSTRACT

Some empirical relations between diffusion characteristics are investigated by the use of carefully examined data from dye release experiments in the surface layer of the sea. These data cover a time scale of diffusion ranging from 1 hour to 1 month and a length scale from 100 m to 100 km. Two "oceanic diffusion diagrams" are prepared; one showing horizontal variance versus diffusion time and the other showing apparent diffusivity versus the scale of diffusion. The overall behaviors of the horizontal variance and of the apparent diffusivity are evidently different from those which the similarity theory of turbulence deduces. However, there still remains a possibility that the similarity theory may be valid locally in time- or length-scale with a variable parameter, i.e., the rate of turbulent energy transfer. The diagrams provide a practical means to predict the rate of horizontal spread of substance introduced from an instantaneous point-source as well as the apparent diffusivity as a function of the size of diffusing patch.

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